THEORY TO TACKLE THE PROBLEMS ABOVE:
For more on number theory check the Number Theory Chapter of Math Book: math-number-theory-88376.html

EXPONENTS

Exponents are a "shortcut" method of showing a number that was multiplied by itself several times. For instance, number a multiplied n times can be written as a^n, where a represents the base, the number that is multiplied by itself n times and n represents the exponent. The exponent indicates how many times to multiple the base, a, by itself.

Exponents one and zero:
$a^0 = 1$ Any nonzero number to the power of 0 is 1.
For example: $5^0 = 1$ and $(-3)^0 = 1$
• **Note: the case of 0^0 is not tested on the GMAT.**

$a^1 = a$ Any number to the power 1 is itself.

Powers of zero:
If the exponent is positive, the power of zero is zero: $0^n = 0$, where $n > 0$.

If the exponent is negative, the power of zero (0^n, where $n < 0$) is undefined, because division by zero is implied.

Powers of one:
$1^n = 1$ The integer powers of one are one.

Negative powers:
$a^{-n} = \frac{1}{a^n}$

Powers of minus one:
If n is an even integer, then $(-1)^n = 1$.

If n is an odd integer, then $(-1)^n = -1$.

Operations involving the same exponents:
Keep the exponent, multiply or divide the bases
$a^n * b^n = (ab)^n$

$\frac{a^n}{b^n} = (\frac{a}{b})^n$

$(a^m)^n = a^{mn}$

$a^{mn} = a^{(m^n)}$ and not $(a^m)^n$ (if exponentiation is indicated by stacked symbols, the rule is to work from the top down)

Operations involving the same bases:
Keep the base, add or subtract the exponent (add for multiplication, subtract for division)
$a^n * a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^{n-m}$

Fraction as power:
$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

ROOTS

Roots (or radicals) are the "opposite" operation of applying exponents. For instance x^2=16 and square root of 16=4.

General rules:

. $\sqrt{x}\sqrt{y}=\sqrt{xy}$ and $\frac{\sqrt{x}}{\sqrt{y}}=\sqrt{\frac{x}{y}}$.

. $(\sqrt{x})^n=\sqrt{x^n}$

. $x^{\frac{1}{n}}=\sqrt[n]{x}$

. $x^{\frac{n}{m}}=\sqrt[m]{x^n}$

. $\sqrt{a}+\sqrt{b}\neq\sqrt{a+b}$

. $\sqrt{x^2}=|x|$, when $x\leq 0$, then $\sqrt{x^2}=-x$ and when $x\geq 0$, then $\sqrt{x^2}=x$

• When the GMAT provides the square root sign for an even root, such as $\sqrt{x}$ or $\sqrt[4]{x}$, then the only accepted answer is the positive root.

That is, $\sqrt{25}=5$, NOT +5 or -5. In contrast, the equation $x^2=25$ has TWO solutions, +5 and -5. **Even roots have only a positive value on the GMAT.**

• Odd roots will have the same sign as the base of the root. For example, $\sqrt[3]{125}=5$ and $\sqrt[3]{-64}=-4$.

1. What is the value of $\sqrt{25+10\sqrt{6}}+\sqrt{25-10\sqrt{6}}$?

A. $2\sqrt{5}$
B. $\sqrt{55}$
C. $2\sqrt{15}$
D. 50
E. 60

2. What is the units digit of $(17^3)^4-1973^2$?

A. 0
B. 2
C. 4
D. 6
E. 8

3. If $5^{10x}=4{,}900$ **and** $2^{\sqrt{y}}=25$ **what is the value of** $\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$ **?**

A. 14/5
B. 5
C. 28/5
D. 13
E. 14

4. What is the value of $5+4*5+4*5^2+4*5^3+4*5^4+4*5^5$?

A. 5^6
B. 5^7
C. 5^8
D. 5^9
E. 5^10

5. If $x = 23^{2}*25^{4}*27^{6}*29^{8}$ and is a multiple of 26^{n}, where n is a non-negative integer, then what is the value of $n^{26}-26^{n}$?

A. -26
B. -25
C. -1
D. 0
E. 1

6. If $x = \sqrt[5]{-37}$ then which of the following must be true?

A. $\sqrt{-x}>2$
B. x>-2
C. x^2<4
D. x^3<-8
E. x^4>32

7. If $x = \sqrt{10}+\sqrt[3]{9}+\sqrt[4]{8}+\sqrt[5]{7}+\sqrt[6]{6}+\sqrt[7]{5}+\sqrt[8]{4}+\sqrt[9]{3}+\sqrt[10]{2}$, then which of the following must be true:

A. x<6
B. 6<x<8
C. 8<x<10
D. 10<x<12
E. x>12

8. If x is a positive number and equals to $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$, where the given expression extends to an infinite number of roots, then what is the value of x?

A. $\sqrt{6}$
B. 3
C. $1+\sqrt{6}$
D. $2\sqrt{3}$
E. 6

9. If x is a positive integer then the value of $\frac{22^{22x}-22^{2x}}{11^{11x}-11^{x}}$ is closest to which of the following?

A. 2^{11x}
B. 11^{11x}
C. 22^{11x}
D. $2^{22x}*11^{11x}$
E. $2^{22x}*11^{22x}$

10. Given that $5x = 125-3y+z$ and $\sqrt{5x}-5-\sqrt{z-3y} = 0$, then what is the value of $\sqrt{\frac{45(z-3y)}{x}}$?

A. 5
B. 10
C. 15
D. 20
E. Can not be determined

11. If $x>0$, $x^{2} = 2^{64}$ and $x^{x} = 2^{y}$ then what is the value of y?

A. 2
B. 2^(11)
C. 2^(32)
D. 2^(37)
E. 2^(64)